Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Thursday 20 June 2019

Morning (Time: 1 hour 30 minutes) | Paper Reference **9FM0/3C**

Further Mathematics

Advanced

Paper 3C: Further Mechanics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

P 6 2 6 7 4 A 0 1 2 8

Pearson

1.

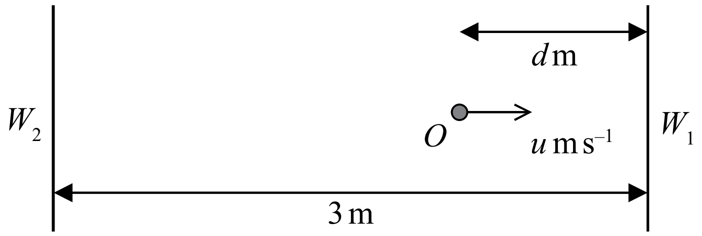


Figure 1

Figure 1 represents the plan of part of a smooth horizontal floor, where W_1 and W_2 are two fixed parallel vertical walls. The walls are 3 metres apart.

A particle lies at rest at a point O on the floor between the two walls, where the point O is d metres, $0 < d \leqslant 3$, from W_1

At time $t = 0$, the particle is projected from O towards W_1 with speed u m s^{-1} in a direction perpendicular to the walls.

The coefficient of restitution between the particle and each wall is $\frac{2}{3}$

The particle returns to O at time $t = T$ seconds, having bounced off each wall once.

(a) Show that $T = \dfrac{45 - 5d}{4u}$

(6)

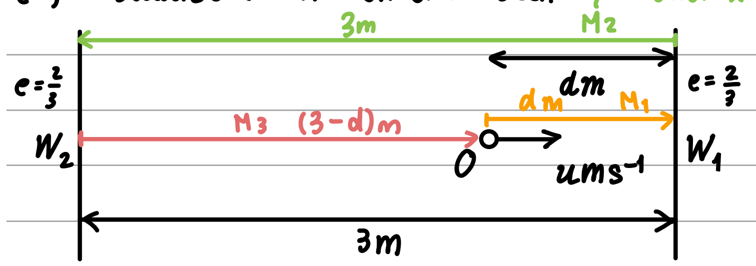
The value of u is fixed, the particle still hits each wall once but the value of d can now vary.

(b) Find the least possible value of T, giving your answer in terms of u. You must give a reason for your answer.

(2)

Question 1 continued

(a) Visualize the motion described:

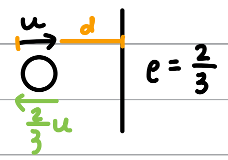


We can use Newton's Law of Restitution to get an equation.
Newton's Law of Restitution states that: when two objects collide, their speeds after the collision depend on ① speeds before the collision and ② the material from which they're made.

Formula: $e(u_A - u_B) = v_B - v_A$

coefficient of restitution — initial speed — final speed

First Collision →

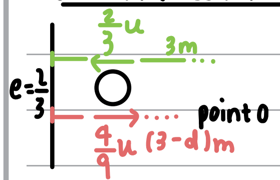


the direction changed!

$\frac{2}{3}(u - 0) = -\frac{2}{3}u$ ‿ speed for motion 2 B1

speed for motion 1

Second Collision



$\frac{2}{3}\left(-\frac{2}{3}u\right) = -\left(-\frac{4}{9}u\right) = \frac{4}{9}u$ ⌒ speed for motion 3 B1

We know that $\text{speed} = \frac{\text{distance}}{\text{time}}$

Apply that to our 3 phases of motion:

M1: d m, t_1 s, u ms^{-1}

M2: 3 m, t_2 s, $\frac{2}{3}u$ ms^{-1} ‿ for speed we don't include signs. speed is scalar

M3: $(3-d)$ m, t_3 s, $\frac{4}{9}u$ ms^{-1}

→ let T be total time: $T = t_1 + t_2 + t_3$ M1

Make t_1, t_2, t_3 the subject: $u = \frac{d}{t_1} \longrightarrow t_1 = \frac{d}{u}$, $\frac{2}{3}u = \frac{3}{t_2} \longrightarrow t_2 = \frac{3}{\frac{2}{3}u}$, $\frac{4}{9}u = \frac{3-d}{t_3} \longrightarrow t_3 = \frac{3-d}{\frac{4}{9}u}$

A1A1

$T = \frac{d}{u} + \frac{3}{\frac{2}{3}u} + \frac{3-d}{\frac{4}{9}u} = \frac{d}{u} + \frac{9}{2u} + \frac{9(3-d)}{4u}$ — make common denominator → $= \frac{4d + 18 + 9(3-d)}{4u}$

$T = \frac{4d + 18 + 9(3-d)}{4u}$ $\xrightarrow{\text{simplify}}$ $T = \frac{4d + 18 + 27 - 9d}{4u} = \frac{45 - 5d}{4u} = T$ hence shown

A1

Question 1 continued

(b) T is minimum when d is maximum. B1

looking at $T = \frac{45 - 5d}{4u}$ this is constant

$d_{max} = 3 \longrightarrow T = \frac{45-15}{4u} = \frac{30}{4u} = \frac{15}{2u}$

$\therefore T_{min} = \frac{15}{2u}$ B1

Question 1 continued

(Total for Question 1 is 8 marks)

2.

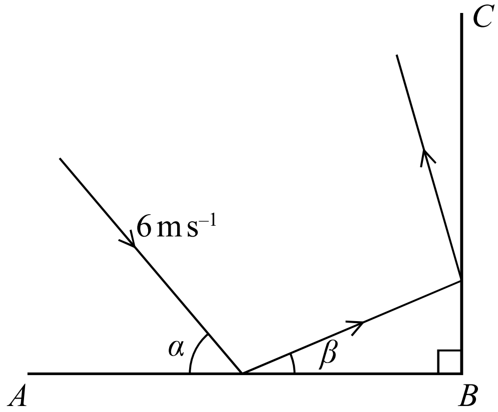


Figure 2

Figure 2 represents the plan view of part of a horizontal floor, where AB and BC are fixed vertical walls with AB perpendicular to BC.

A small ball is projected along the floor towards AB with speed $6\,\text{m s}^{-1}$ on a path that makes an angle α with AB, where $\tan\alpha = \frac{4}{3}$. The ball hits AB and then hits BC. Immediately after hitting AB, the ball is moving at an angle β to AB, where $\tan\beta = \frac{1}{3}$

The coefficient of restitution between the ball and AB is e.

The coefficient of restitution between the ball and BC is $\frac{1}{2}$

By modelling the ball as a particle and the floor and walls as being smooth,

(a) show that the value of $e = \frac{1}{4}$

(5)

(b) find the speed of the ball immediately after it hits BC.

(4)

(c) Suggest two ways in which the model could be refined to make it more realistic.

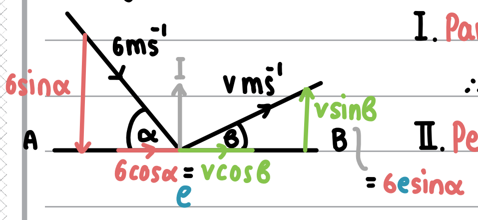
(2)

Question 2 continued

(a) We need to consider the collision with AB

→ Method 1 use trigonometric expressions

Diagram



I. Parallel to the wall, the velocity doesn't change

$\therefore \quad 6\cos\alpha = v\cos\beta$ B1

II. Perpendicular to the wall, use newton's law of restitution.

$e \times 6\sin\alpha = v\sin\beta$ M1 A1

we know that $\tan\beta = \frac{1}{3}$.

Using trigonometry, we know that $\tan\beta = \frac{\sin\beta}{\cos\beta}$

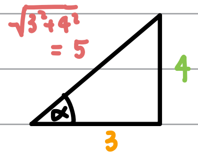
$$\tan\beta = \frac{\cancel{v}\sin\beta}{\cancel{v}\cos\beta} = \frac{\cancel{6}e\sin\alpha}{\cancel{6}\cos\alpha} \quad \text{M1}$$

$$\tan\beta = e\tan\alpha$$

we know that $\tan\alpha = \frac{4}{3}$:

$$\therefore \frac{1}{3} = e \times \frac{4}{3} \Rightarrow e = \frac{1}{4} \quad \text{value of } e \quad \text{A1}$$

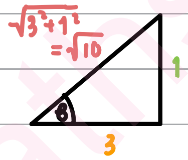
→ Method 2 use numerical values for the trigonometric expressions



$\tan\alpha = \frac{O}{A} = \frac{4}{3}$ given

$\sin\alpha = \frac{O}{H} = \frac{4}{5}$

$\cos\alpha = \frac{A}{H} = \frac{3}{5}$

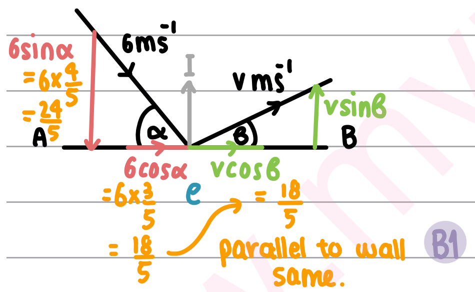


$\tan\beta = \frac{O}{A} = \frac{1}{3}$ given

$\sin\beta = \frac{O}{H} = \frac{1}{\sqrt{10}}$

$\cos\beta = \frac{A}{H} = \frac{3}{\sqrt{10}}$

Diagram



Perpendicular to the wall, use newton's law of restitution.

$e \times 6\sin\alpha = v\sin\beta$

$e \times \frac{24}{5} = \frac{24}{5}e$ M1A1

Way 1: Use fact that $\tan\beta = \frac{1}{3}$.

$\tan\beta = \frac{O}{H}$ $\quad$ $\tan\beta = \frac{\sin\beta}{\cos\beta}$

$v\sin\beta$, $v\cos\beta$

$$\frac{\frac{24}{\cancel{5}}e}{\frac{18}{\cancel{5}}} = \tan\beta = \frac{1}{3} \quad \text{M1}$$

$$\frac{24}{18}e = \frac{1}{3}$$

$$e = \frac{\cancel{18}^{6}}{\cancel{3}\times 24} = \frac{6}{24} = \frac{1}{4} \quad \therefore e = \frac{1}{4} \quad \text{value of } e \quad \text{A1}$$

Question 2 continued

Way 2: velocity components

I. Perpendicular to the wall:

Rearrange $6e\sin\alpha = v\sin\beta$

$$e = \frac{v\sin\beta}{6\sin\alpha} \qquad \text{Eq.1}$$

II. Parallel to the wall

$$6\cos\alpha = v\cos\beta$$

$$v = \frac{6\cos\alpha}{\cos\beta}$$

Substitute $v = \ldots$ into Eq.1

$$\tan\beta = \frac{\sin\beta}{\cos\beta}$$

$$e = \frac{\left(\frac{6\cos\alpha}{\cos\beta}\right)\sin\beta}{6\sin\alpha} = \frac{\cancel{6}\cos\alpha\tan\beta}{\cancel{6}\sin\alpha} = \tan\beta \times \frac{1}{\tan\alpha} \quad \text{M1}$$

We know that $\tan\beta = \frac{1}{3}$ and $\tan\alpha = \frac{4}{3}$

Substitute into

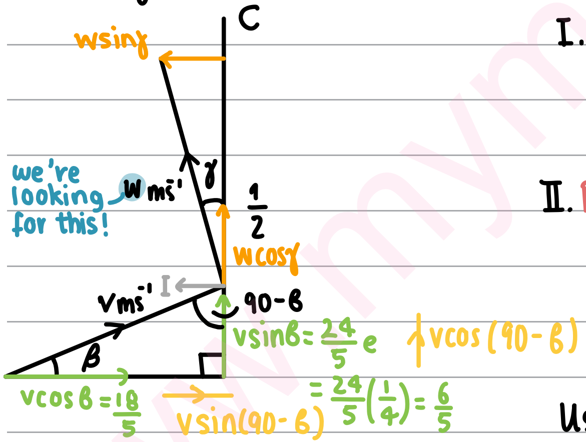
$$e = \tan\beta \times \frac{1}{\tan\alpha}$$

$$e = \frac{1}{3} \times \frac{1}{\frac{4}{3}} = \frac{1}{3} \times \frac{3}{4} = \frac{1}{4}$$

$$\therefore e = \frac{1}{4} \quad \text{value of } e \quad \text{A1}$$

(b) We need to consider the collision with BC

Diagram



Method 1 - use angle β

I. Parallel to the wall velocity doesn't change

$$v\sin\beta = w\cos\gamma$$

$$\text{B1} \quad \frac{6}{5} = w\cos\gamma \quad \text{parallel comp.}$$

II. Perpendicular to the wall - Newton's law of Restitution

$$e v\cos\beta = w\sin\gamma$$

$$\frac{1}{2}\left(\frac{\cancel{18}^{\,9}}{5}\right) = w\sin\gamma$$

$$\text{B1} \quad \frac{9}{5} = w\sin\gamma \quad \text{perpendicular comp.}$$

Use Pythagoras' Theorem to get the speed, $|w|$.

$$|w| = \sqrt{\text{perp}^2 + \text{paral}^2}$$

$$|w| = \sqrt{\left(\frac{9}{5}\right)^2 + \left(\frac{6}{5}\right)^2} \quad \text{M1}$$

$$= \frac{3\sqrt{13}}{5}\ \text{ms}^{-1} \quad \text{speed } w \quad \text{A1}$$

Question 2 continued

yellow labels!

Method 2 - use angle $(90-\beta)$

I. Parallel to the wall velocity doesn't change

$v\cos(90-\beta) = w\cos\gamma$ (angle properties: $\cos(90-\beta) = \sin\beta$)

$v\sin\beta = w\cos\gamma \longrightarrow \frac{6}{5} = w\cos\gamma$ parallel comp. B1

II. Perpendicular to the wall - Newton's law of Restitution

$ev\sin(90-\beta) = w\sin\gamma$ (angle properties: $\sin(90-\beta) = \cos\beta$)

$ev\cos\beta = w\sin\gamma \longrightarrow e \times \frac{18}{5} = \frac{1}{2} \times \frac{18}{5} = \frac{9}{5} = w\sin\gamma$ perpendicular comp. B1

Use Pythagoras' Theorem to get the speed, $|w|$.

$|w| = \sqrt{\text{perp}^2 + \text{paral}^2}$

$|w| = \sqrt{\left(\frac{9}{5}\right)^2 + \left(\frac{6}{5}\right)^2}$ M1

$= \frac{3\sqrt{13}}{5}\ \text{ms}^{-1}$ speed w A1

(c) Give the ball dimension

Consider air resistance

Consider the spin of the ball B1 B1

(Total for Question 2 is 11 marks)

3. A particle P, of mass 0.5 kg, is moving with velocity $(4\mathbf{i} + 4\mathbf{j})\,\mathrm{m\,s^{-1}}$ when it receives an impulse $\mathbf{I}$ of magnitude 2.5 Ns.

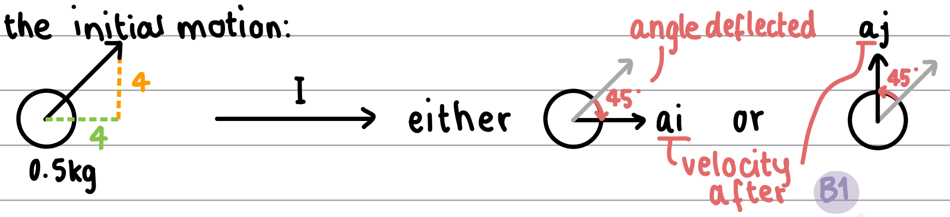
As a result of the impulse, the direction of motion of P is deflected through an angle of 45°

Given that $\mathbf{I} = (\lambda\mathbf{i} + \mu\mathbf{j})\,\mathrm{Ns}$, find all the possible pairs of values of λ and μ.

(9)

Method 1 – use final velocity

Visualize the initial motion:



Impulse is the change in momentum

Formula for change in momentum:

$$I = \Delta\text{momentum} = m v_{\text{final}} - m v_{\text{initial}}$$

mass

initial velocity

We can use $I = mv - mu = m(v-u)$ to get an equation:

$$(\lambda i + \mu j) = \frac{1}{2}(ai - (4i+4j))$$ initial velocity

$$= \frac{1}{2}((a-4)i - 4j)$$

$$(\lambda i + \mu j) = (\frac{a}{2} - 2)i - 2j$$ M1

We are given that $I = (\lambda i + \mu j)\,\mathrm{Ns}$ and $|I| = 2.5\,\mathrm{Ns}$

So we can use the Pythagoras' theorem to get an equation for μ and λ:

$$\frac{5}{2} = \sqrt{\lambda^2 + \mu^2}$$ M1

$$\therefore \lambda^2 + \mu^2 = \frac{25}{4}$$

Now we can substitute from $(\lambda i + \mu j) = (\frac{a}{2} - 2)i - 2j$:

$$\lambda = \frac{a}{2} - 2$$

$$\mu = -2$$

$$(\frac{a}{2} - 2)^2 + (-2)^2 = \frac{25}{4}$$ A1

$$\frac{a^2}{4} - 2a + 4 + 4 - \frac{25}{4} = 0$$

$$\frac{a^2}{4} - 2a + \frac{7}{4} = 0 \quad |\times 4| \quad a^2 - 8a + 7 = 0$$ factorize A1

$$(a-7)(a-1) = 0$$

$$a = 7,\ a = 1$$ → possible final velocities

So the first pair is $\lambda = \frac{7}{2} - 2 = \frac{3}{2}$, $\mu = -2$ → $\lambda = \frac{3}{2}, \mu = -2$ $\quad I = (\frac{3}{2}i - 2j)\,\mathrm{Ns}$ M1

the second pair is $\lambda = \frac{1}{2} - 2 = -\frac{3}{2}$, $\mu = -2$ → $\lambda = -\frac{3}{2}, \mu = -2$ $\quad I = (-\frac{3}{2}i - 2j)\,\mathrm{Ns}$ A1

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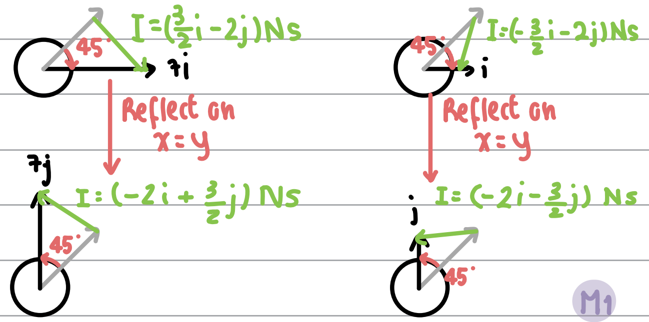
Question 3 continued

Now let's draw it out and use symmetry:

For the case it turns towards j:



M1

∴ the four pairs of λ and μ:

$\boxed{I = (\tfrac{3}{2}i - 2j)\,Ns}$

$\boxed{I = (-\tfrac{3}{2}i - 2j)\,Ns}$

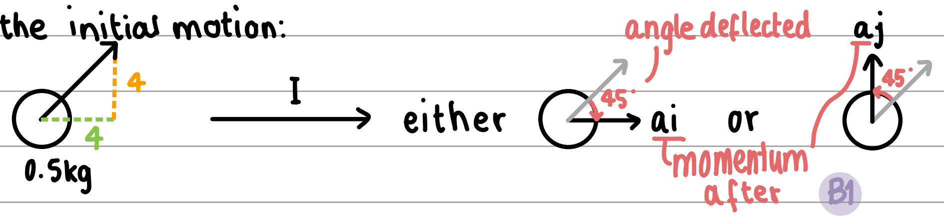
$\boxed{I = (-2i + \tfrac{3}{2}j)\,Ns}$

$\boxed{I = (-2i - \tfrac{3}{2}j)\,Ns}$ A1

Question 3 continued

Method 2 - use final momentum

Visualize the initial motion:



Impulse is the change in momentum

Formula for change in momentum:

$$I = \Delta\text{momentum} = \overset{\text{mass}}{m}v_{\text{final}} - mv_{\text{initial}}$$

(velocity)

Substitute to get an equation:

$I = m(v-u) \longrightarrow (\lambda i + \mu j) = \frac{1}{2}(2ai - \overset{u}{(4i+4j)}) = \frac{1}{2}(2a-4)i - \frac{1}{2}(4)j = (a-2)i - 2j$

for the momentum to be ai: M1

$p = mv,\ m = \frac{1}{2}\ \therefore\ a = \frac{1}{2}v,\ v = 2a$

We are given that $I = (\lambda i + \mu j)$ Ns and $|I| = 2.5$ Ns

So we can use the Pythagoras' theorem to get an equation for μ and λ: M1

$$\frac{5}{2} = \sqrt{\lambda^2 + \mu^2}$$

$$\therefore \lambda^2 + \mu^2 = \frac{25}{4}$$

Substitute in our I:

$$(a-2)^2 + (-2)^2 = \frac{25}{4}$$

$$a^2 - 4a + 4 + 4 = \frac{25}{4}$$

$$a^2 - 4a + 8 - \frac{25}{4} = 0$$

$$a^2 - 4a + \frac{7}{4} = 0$$

A1 $4a^2 - 16a + 7 = 0$ factorize to solve for a

$$(2a-7)(2a-1) = 0$$

$a = \frac{7}{2}\quad a = \frac{1}{2}$ A1

Substitute our values of a back into our $I = (a-2)i - 2j$:

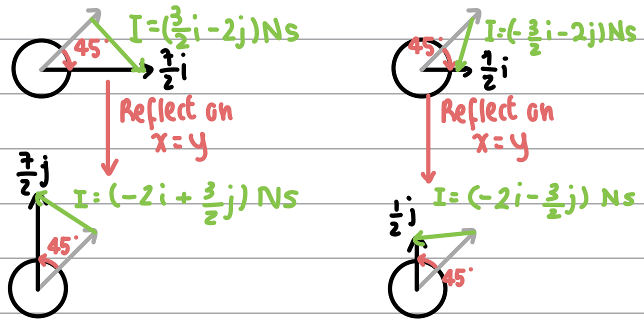
$a = \frac{7}{2}$: $\quad I = (\frac{7}{2} - 2)i - 2j = \frac{3}{2}i - 2j$ M1

$a = \frac{1}{2}$: $\quad I = (\frac{1}{2} - 2)i - 2j = -\frac{1}{2}i - 2j$ A1

Question 3 continued

Now let's draw it out and use symmetry:

For the case it turns towards j:



M1

∴ the four pairs of λ and μ:

$I = (\frac{3}{2}i - 2j)$ Ns

$I = (-\frac{3}{2}i - 2j)$ Ns

$I = (-2i + \frac{3}{2}j)$ Ns

$I = (-2i - \frac{3}{2}j)$ Ns A1

(Total for Question 3 is 9 marks)

4. A car of mass 600 kg pulls a trailer of mass 150 kg along a straight horizontal road. The trailer is connected to the car by a light inextensible towbar, which is parallel to the direction of motion of the car. The resistance to the motion of the trailer is modelled as a constant force of magnitude 200 N. At the instant when the speed of the car is v m s^{-1}, the resistance to the motion of the car is modelled as a force of magnitude $(200 + \lambda v)$ N, where λ is a constant.

When the engine of the car is working at a constant rate of 15 kW, the car is moving at a constant speed of 25 m s^{-1}

(a) Show that $\lambda = 8$

(4)

Later on, the car is pulling the trailer up a straight road inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{15}$

The resistance to the motion of the trailer from non-gravitational forces is modelled as a constant force of magnitude 200 N at all times. At the instant when the speed of the car is v m s^{-1}, the resistance to the motion of the car from non-gravitational forces is modelled as a force of magnitude $(200 + 8v)$ N.

The engine of the car is again working at a constant rate of 15 kW.

When $v = 10$, the towbar breaks. The trailer comes to instantaneous rest after moving a distance d metres up the road from the point where the towbar broke.

distance

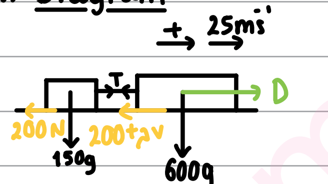
(b) Find the acceleration of the car immediately after the towbar breaks.

(4)

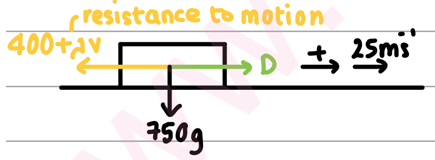
(c) Use the work-energy principle to find the value of d.

(4)

(a) Diagram



We will treat the car + trailer as a system:



Since we are given that the speed is constant, use $\Sigma F_x = 0$: M1

$D = 400 + \lambda v$ A1

Substitute $v = 25 ms^{-1}$:

$D = 400 + 25\lambda$

To get D we will use Power.

Formula for Power:

Power (W) — $P = D v$

Driving force (N) velocity (ms⁻¹)

$P = 15kW \xrightarrow{\times 1000} 15000W$

$D = D$

$v = 25 ms^{-1}$

Substitute: $15000 = 25D$

$D = \frac{15000}{25} = 600N$ B1

Substitute back into $D = 400 + 25\lambda \longrightarrow 600 = 400 + 25\lambda$

$200 = 25\lambda$

A1 $\lambda = 8$ hence shown

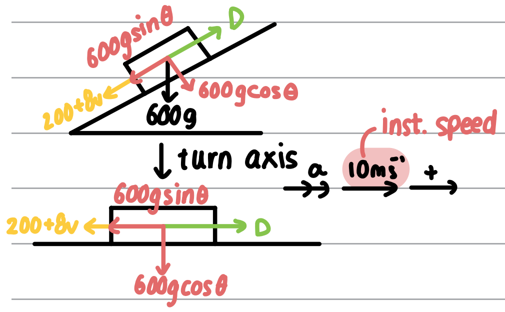
Question 4 continued

(b) Diagram

Before towbar breaks

After towbar breaks

☆ for simplicity omit the trailer



Since we're looking for acceleration, use $\Sigma F_x = ma$ $(\overset{+}{\rightarrow})$

$D - 600g\sin\theta - (200+8v) = 600a$ M1A1

To get D we will use Power.

Formula for Power:

Power (W) $P = Dv$ — Driving force (N), velocity (ms^{-1})

$P = 15kW \xrightarrow{\times 1000} 15\,000W$

$D = D$

$v = 10ms^{-1}$

$15\,000 = 10D$

$D = 1500N$

Substitute:

$1500 - \cancel{600}^{40}g \times \frac{1}{\cancel{15}} - (200+8(10)) = 600a$ M1

$1500 - 40g - 280 = 600a$

$\frac{122\cancel{0} - 4\cancel{0}g}{60\cancel{0}} = a$

$\boxed{a = 1.38ms^{-2} \text{ to } 3sf}$ A1

Question 4 continued

(c)

★ Work-Energy Principle: an increase of KE/GPE/EPE is caused by an equal amount of positive work done on the body (e.g. engine) and a decrease of KE/GPE/EPE is caused by an equal amount of negative work done on the body (e.g. friction).

★ Remember the work-energy formulae:

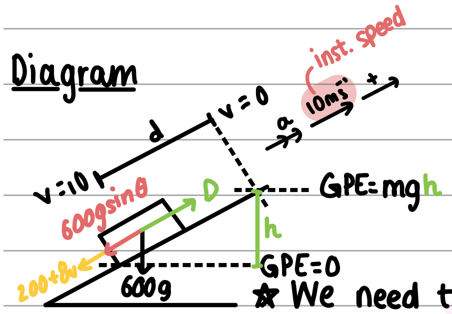
Either: $\underbrace{\text{WD by force}}_{\text{work done}} + \underbrace{KE_i}_{\text{initial kinetic}} + \underbrace{GPE_i}_{\text{initial grav. potential}} = \underbrace{KE_f}_{\text{final kinetic}} + \overbrace{GPE_f}^{\text{final grav. potential}} + \underbrace{\text{WD against friction}}_{\text{work lost to friction}}$

OR: $\underbrace{\text{WD by force}}_{\text{work done}} + \underbrace{KE_i}_{\text{initial kinetic}} + \underbrace{GPE_i}_{\text{initial grav. potential}} - \underbrace{\text{WD by friction}}_{\text{we subtract this since it leaves the system as heat!}} = \underbrace{KE_f}_{\text{final kinetic}} + \overbrace{GPE_f}^{\text{final grav. potential}}$ M1

★ Formulae for KE and GPE:

$KE = \frac{1}{2}\underbrace{m}_{\text{mass}}\underbrace{v^2}_{\text{velocity}}$ $\qquad GPE = \underbrace{m}_{\text{mass}}\underbrace{g}_{g=9.8ms^{-2}}\underbrace{h}_{\text{change in height}}$

Diagram



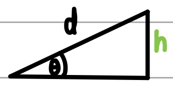
★ We need to set GPE=0 so we can have a reference point.

Substitute:

$\frac{1}{2}(600)(10)^2 + \cancel{mgh}^{0} - dF = \cancel{\frac{1}{2}(600)(0)}^{0} + mgh$ A1

$300(100) - d(200 + 8(10)) = mgh$

To get h:



$\sin\theta = \frac{1}{15} = \frac{h}{d}$ $\qquad h = \frac{d}{15}$

Substitute h and solve for d:

$30000 - 280d = \overset{40}{\cancel{600}}g\left(\frac{d}{\cancel{15}}\right)$ A1

$30000 = 40gd + 280d$

$30000 = d(40g + 280)$

$d = \frac{3000\cancel{0}}{(4\cancel{0}g + 28\cancel{0})}$ $\qquad \boxed{d = 25.2m \text{ to 3sf}}$ A1

Question 4 continued

(Total for Question 4 is 12 marks)

5. A particle P of mass $3m$ and a particle Q of mass $2m$ are moving along the same straight line on a smooth horizontal plane. The particles are moving in opposite directions towards each other and collide directly.

Immediately before the collision the speed of P is u and the speed of Q is $2u$.

Immediately after the collision P and Q are moving in opposite directions.

both change direction

The coefficient of restitution between P and Q is e.

(a) Find the range of possible values of e, justifying your answer.

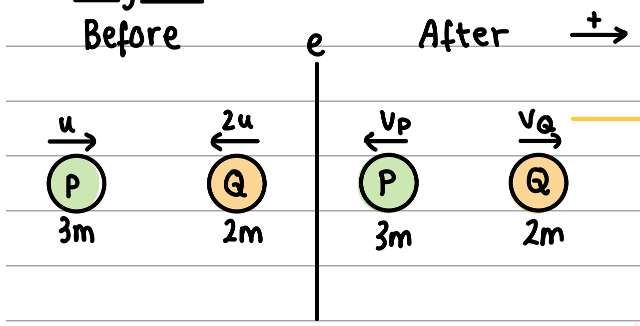
(8)

Given that Q loses 75% of its kinetic energy as a result of the collision,

(b) find the value of e.

(3)

(a) Diagram



we are given that both directions changed, so we reverse the direction on the diagram. for this to happen, we'll want our v_Q (and our v_P) to be positive, since we already assume the right direction in our diagram

We can use the conservation of linear momentum to get this.

conservation of linear momentum means: the total momentum before the collision is the same as the total momentum after.

Formula: $m_A u_A + m_B u_B = m_A v_A + m_B v_B$ M1

initial velocity final velocity

Substitute:

pay attention to the direction

$3m(u) + 2m(-2u) = 3m(-V_P) + 2m(V_Q)$ cancel m's

$3u - 4u = -3v_P + 2v_Q$

$-u = -3v_P + 2v_Q$ Eq1 A1

We can use Newton's Law of Restitution to get an equation.

Newton's Law of Restitution states that: when two objects collide, their speeds after the collision depend on ① speeds before the collision and ② the material from which they're made.

Formula: $e(u_A - u_B) = v_B - v_A$ M1

coefficient of restitution initial speed final speed

Substitute: $e(u-(-2u)) = v_Q - -v_P$

$3eu = v_Q + v_P$ Eq2 A1

P 6 2 6 7 4 A 0 1 8 2 8

Question 5 continued

Simultaneous equations Eq1 and Eq2:

$$-u = -3v_P + 2v_Q \quad \text{Eq1}$$

$$3eu = v_Q + v_P \quad \text{Eq2}$$

Use elimination method to get an equation for v_Q

$$-u = -3v_P + 2v_Q \quad +$$

$$9eu = 3v_P + 3v_Q$$

$$9eu - u = 5v_Q$$

$$v_Q = \frac{u}{5}(9e-1) \quad \text{M1}$$

$v_Q > 0$ for it to move in the correct direction.

$$\frac{u}{5}(9e-1) > 0$$

$$9e - 1 > 0$$

$$e > \frac{1}{9} \quad \text{A1}$$

Get equation for v_P:

$$-u = -3v_P + 2v_Q \quad -$$

$$6eu = 2v_P + 2v_Q$$

$$-6eu - u = -5v_P$$

$$6eu + u = 5v_P$$

$$\frac{u}{5}(6e+1) = v_P \quad v_P > 0 \text{ for all } e \quad \text{M1}$$

Since e is a coefficient of restitution, it must be smaller than 1.

∴ complete answer:

$$\frac{1}{9} < e \leq 1 \quad \text{A1}$$

(b) We are given that: $KE_{final} = \frac{25}{100} KE_{initial}$ M1

Formula for Kinetic Energy:

$$KE = \frac{1}{2} m v^2$$

mass velocity

For Q:

$v_{init} = 2u$

$v_{final} = \frac{u}{5}(9e-1)$

$m = 2m$

Substitute:

$$\frac{1}{\cancel{2}}(\cancel{2m})\left(\frac{u}{5}(9e-1)\right)^2 = \frac{25}{100}\left(\frac{1}{2}\right)(2\cancel{m})(2u)^2 \quad \text{A1}$$

$$\left(\frac{u(9e-1)}{5}\right)^2 = \frac{1}{\cancel{4}} \times \frac{1}{\cancel{2}}(\cancel{2})(\cancel{4}u^2)$$

$$\frac{\cancel{u^2}(9e-1)^2}{25} = \cancel{u^2}$$

$$(9e-1)^2 = 25$$

$$9e - 1 = \pm 5$$

$9e - 1 = 5 \longrightarrow 9e = 4 \quad e = \frac{4}{9}$

$9e - 1 = -5 \quad 9e = -4 \quad \cancel{e = -\frac{4}{9}}$ Reject as $0 < e \leq 1$

$$e = \frac{2}{3} \text{ value of } e \quad \text{A1}$$

Question 5 continued

Question 5 continued

(Total for Question 5 is 11 marks)

6. [*In this question* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

A smooth uniform sphere A has mass 0.2 kg and another smooth uniform sphere B, with the same radius as A, has mass 0.4 kg.

The spheres are moving on a smooth horizontal surface when they collide obliquely. Immediately before the collision, the velocity of A is $(3\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$ and the velocity of B is $(-4\mathbf{i} - \mathbf{j})\,\text{m s}^{-1}$

At the instant of collision, the line joining the centres of the spheres is parallel to **i**

The coefficient of restitution between the spheres is $\frac{3}{7}$

(a) Find the velocity of A immediately after the collision.

(7)

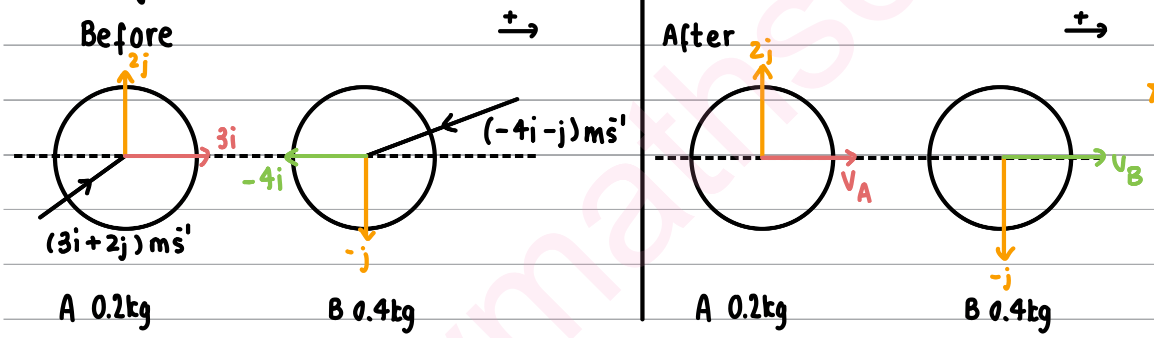
(b) Find the magnitude of the impulse received by A in the collision.

(2)

(c) Find, to the nearest degree, the size of the angle through which the direction of motion of A is deflected as a result of the collision.

(3)

(a) Diagram $e = \frac{3}{7}$



★perpendicular components stay the same! B1

→ We are looking for v_A.

We can use the conservation of linear momentum on the parallel components.

conservation of linear momentum means: the total momentum before the collision is the same as the total momentum after. M1

Formula: $m_A \underbrace{u_A}_{\text{initial velocity}} + m_B u_B = m_A \underbrace{v_A}_{\text{final velocity}} + m_B v_B$

Substitute:

$$0.2(3) + 0.4(-4) = 0.2v_A + 0.4v_B$$

$$-1 = 0.2v_A + 0.4v_B$$

$$-5 = v_A + 2v_B \quad \text{Eq. 1} \quad \text{A1}$$

Question 6 continued

We can use Newton's Law of Restitution on the parallel components. M1

Newton's Law of Restitution states that: when two objects collide, their speeds after the collision depend on ① speeds before the collision and ② the material from which they're made.

Formula: $e(u_A - u_B) = v_B - v_A$

coefficient of restitution, initial speed, final speed

Substitute:

$$\frac{3}{7}(3-(-4)) = v_B - v_A$$

$$3 = v_B - v_A \quad \text{Eq2} \quad \text{A1}$$

Solve simultaneously Eq1 and Eq2.

$$-5 = v_A + 2v_B$$

$$v_A + 3 = v_B$$

$$-5 = v_A + 2(v_A + 3)$$

$$-5 = v_A + 2v_A + 6$$

$$-11 = 3v_A \quad v_A = -\frac{11}{3} \quad \text{M1}$$

∴ the velocity of A after:

$$\left(-\frac{11}{3}\mathbf{i} + 2\mathbf{j}\right)\text{ms}^{-1} \quad \text{A1}$$

(b) Impulse is the change in momentum

Formula for change in momentum:

$$\Delta\text{momentum} = mv_{final} - mv_{initial}$$

mass, velocity

Since only the parallel components change we can just apply the formula to those:

$$I = m(v-u)$$

$$\text{M1} \quad = 0.2\left(\frac{11}{3} - (-3)\right)$$ take ⟵+ as positive here

$$= 0.2\left(\frac{11}{3} + 3\right)$$

$$= \frac{4}{3}\text{ Ns} \quad \text{A1}$$

(c) We will use the scalar product formula:

$$\cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|\times|\mathbf{b}|} \quad \text{M1}$$

Remember dot product: $\begin{pmatrix} a \\ b \end{pmatrix} \cdot \begin{pmatrix} c \\ d \end{pmatrix} = ac + bd$

Substitute:

$$\cos^{-1}\left(\frac{\begin{pmatrix} -\frac{11}{3} \\ 2 \end{pmatrix}\cdot\begin{pmatrix} 3 \\ 2 \end{pmatrix}}{\sqrt{\left(-\frac{11}{3}\right)^2 + 2^2} \times \sqrt{3^2+2^2}}\right) = \cos^{-1}\left(\frac{-7}{\sqrt{13}\times\sqrt{\frac{157}{9}}}\right) = \theta \quad \text{A1}$$

$$\theta = 118° \quad \text{A1}$$

Question 6 continued

Question 6 continued

(Total for Question 6 is 12 marks)

7. A particle P, of mass m, is attached to one end of a light elastic spring of natural length a and modulus of elasticity kmg.

The other end of the spring is attached to a fixed point O on a ceiling.

The point A is vertically below O such that $OA = 3a$

The point B is vertically below O such that $OB = \frac{1}{2}a$

The particle is held at rest at A, then released and first comes to instantaneous rest at the point B.

(a) Show that $k = \frac{4}{3}$
(3)

(b) Find, in terms of g, the acceleration of P immediately after it is released from rest at A.
(3)

(c) Find, in terms of g and a, the maximum speed attained by P as it moves from A to B.
(6)

(a) ★ conservation of mechanical energy principle: states that the total amount of mechanical energy (KE/GPE/EPE) in a closed system in the absence of dissipative forces (e.g. friction/air resistance) remains constant.

★ Remember the work-energy formula:

Either: $KE_i + GPE_i + EPE_i = KE_F + GPE_F + EPE_F$

initial kinetic, initial grav. potential, initial elastic potential, final kinetic, final grav. potential, final elastic potential

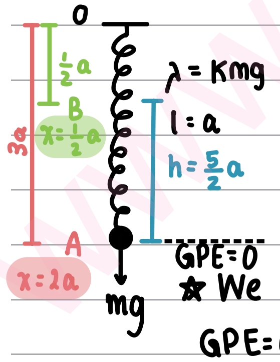
★ Formulae for KE, GPE and EPE:

$KE = \frac{1}{2}mv^2$ (m: mass, v: velocity)

$GPE = mgh$ (m: mass, h: change in height, $g = 9.8\,ms^{-2}$)

$EPE = \frac{\lambda x^2}{2l}$ (λ: modulus of elasticity, x: extension of string/spring, l: natural length of the string/spring)

Diagram



★ We need to set GPE=0 so we can have a reference point.

Substitute:

It moves from rest to rest $\therefore KE_I = KE_F = 0$

$\cancel{GPE_i}^{0} + EPE_i = GPE_F + EPE_F$ M1

$\frac{(kmg)(2a)^2}{2a} = mg\left(\frac{5}{2}a\right) + \frac{(kmg)(\frac{1}{2}a)^2}{2a}$

A1 $\frac{k\cancel{mg}(4a^{\cancel{2}})}{2\cancel{a}} - \frac{k\cancel{mg}(\frac{a^{\cancel{2}}}{4})}{2\cancel{a}} = \frac{5}{2}\cancel{mg}a$ cancel mg's

$\frac{4ak}{2} - \frac{ak}{8} = \frac{5}{2}a$

$2k - \frac{k}{8} = \frac{5}{2}$

$16k - k = 20$

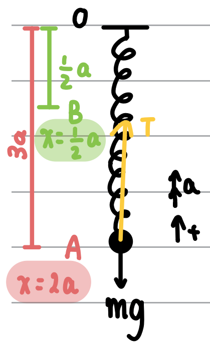
$15k = 20 \longrightarrow k = \frac{20}{15} = \frac{4}{3} \longrightarrow \boxed{k = \frac{4}{3}}$ A1

shown.

P 6 2 6 7 4 A 0 2 6 2 8

Question 7 continued

(b) Diagram



at A:

$\lambda = Kmg$

$l = a$

$x = 2a$

$\uparrow a \quad \uparrow +$

To get T:

Formula for tension in the spring:

$$T = \frac{\lambda x}{l}$$

Substitute:

$$T = \frac{(\frac{4}{3}mg)(2a)}{a}$$

We will use $\Sigma F_y = ma$ as we are looking for acceleration:

$$T - mg = ma \quad \text{M1}$$

$$\frac{4\cancel{m}g \times 2\cancel{a}}{3\cancel{a}} - \cancel{m}g = \cancel{m}a \quad \text{A1}$$

$$\frac{8}{3}g - g = a$$

$$\boxed{\frac{5}{3}g = a} \quad \text{acceleration} \quad \text{A1}$$

(c) The maximum speed is reached at equilibrium position, where $\Sigma F_y = 0$ M1

$$\therefore \; T = mg$$

$$\frac{\lambda x}{l} = mg \quad \text{extension at equil.}$$

$$\frac{\frac{4}{3}\cancel{mg}\, e}{a} = \cancel{mg} \quad \text{cancel } mg\text{'s}$$

$$\boxed{e = \frac{3}{4}a} \quad \text{A1}$$

☆ conservation of mechanical energy principle: states that the total amount of mechanical energy (KE/GPE/EPE) in a closed system in the absence of dissipative forces (e.g. friction/air resistance) remains constant.

★ Remember the work-energy formula:

Either:

$$\underbrace{KE_i}_{\text{initial kinetic}} + \underbrace{GPE_i}_{\text{initial grav. potential}} + \underbrace{EPE_i}_{\text{initial elastic potential}} = \underbrace{KE_F}_{\text{final kinetic}} + \overbrace{GPE_F}^{\text{final grav. potential}} + \underbrace{EPE_F}_{\text{final elastic potential}}$$

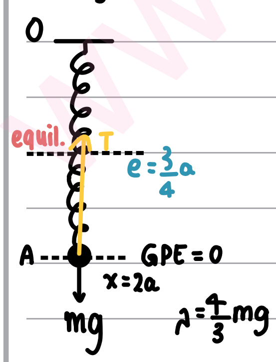
☆ Formulae for KE, GPE and EPE:

$KE = \frac{1}{2}mv^2$ — velocity; mass

$GPE = mgh$ — change in height; mass; $g = 9.8ms^{-2}$

$EPE = \frac{\lambda x^2}{2l}$ — modulus of elasticity; extension of string/spring; natural length of the string/spring

Diagram



Substitute:

$$\cancel{\frac{1}{2}mv^2}^{0} + \cancel{mgh}^{0} + \frac{(\frac{4}{3}mg)(2a)^2}{2a} = \frac{1}{2}mv^2 + mg(2a - e) + \frac{(\frac{4}{3}mg)\,e^2}{2a} \quad \text{M1A1}$$

Solve to make v the subject:

$$\frac{4\cancel{m}g \times 4a^{\cancel{2}}}{6\cancel{a}} = \frac{1}{2}\cancel{m}v^2 + \cancel{m}g(2a - \frac{3}{4}a) + \frac{4\cancel{m}g(\frac{3}{4}a)^2}{6a} \quad \text{cancel } m\text{'s}$$

$$\frac{16}{6}ag = \frac{1}{2}v^2 + \frac{5}{4}ga + \frac{4g(\frac{9}{16}a^2)}{24a}$$

$$\frac{8}{3}ag - \frac{5}{4}ag = \frac{1}{2}v^2 + \frac{3}{8}ag$$

$$\frac{8}{3}ag - \frac{5}{4}ag - \frac{3}{8}ag = \frac{1}{2}v^2$$

$$\frac{25}{24}ag = v^2 \quad \text{A1}$$

$$\boxed{v = \frac{5}{2}\sqrt{\frac{ga}{3}}} \quad \text{A1}$$

Question 7 continued

(Total for Question 7 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS